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GX 5-1: A MODEL FOR THE QUASIPERIODIC OSCILLATIONS

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Quasiperiodic oscillations with frequencies in the range ~ 20-40 Hz have recently been reported[1] in the 1-10 keV flux from the bright galactic-bulge X-ray source GX 5-1. We suggest that GX 5-1 is a weakly magnetic neutron star accreting from a Keplerian disk in which the plasma is clumped. Interaction of the magnetosphere with these large-amplitude spatial variations in the inner disk causes the X-ray flux from the system to be modulated at the beat frequency given by the difference between the rotation frequency of the star and the rotation frequency of the inner disk, or at some harmonic of this beat frequency. This beat frequency and its low harmonics are comparable to the frequency of the oscillations observed in GX 5-1 for surface magnetic fields ~ 10^9 G and accretion rates ~ 10^{18} g s^{-1}.[2] Moreover, the variation of the beat frequency with accretion rate is similar to the variation of the oscillation frequency with X-ray flux reported in GX 5-1, if the neutron star is rotating near its equilibrium spin rate.[3] Red noise in the X-ray flux at frequencies

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below that of the quasiperiodic oscillations is an unavoidable consequence of this model. It follows from the model that the strength of the red noise rises and falls with the strength of the quasi-periodic oscillations, as has been observed in GX 5-1.[4] We suggest why X-ray pulsations at the rotation frequency of the neutron star are difficult to observe. Although motivated by the reported quasiperiodic oscillations in GX 5-1, the model outlined here should be applicable to a wide range of systems. We therefore describe the general features of the model before applying it to GX 5-1.

Where the disk plasma interacts with the magnetic field of the star, a boundary layer forms.[5-7] In this layer, plasma drifts inward as angular momentum is removed from it by viscous transport outward and by interaction with the stellar magnetic field. We assume that there are large-amplitude spatial variations in the density and other properties of a portion of the disk plasma entering this boundary layer. We assume further that the X-ray flux from the system varies with the azimuthal position of each inhomogeneity or 'clump' relative to one or more special directions that rotate with the star. Below, we discuss several examples of specific physical mechanisms that could cause clumping and subsequent modulation of the X-ray flux from the system. First, however, we wish to call attention to the general features of this model, which are insensitive to these details.

General Features of the Model

We define the boundary layer as the annulus at the inner edge of the disk containing those clumps that contribute to the oscillating X-ray flux. As a given clump orbits within the boundary layer, it periodically intersects its special direction(s), which are fixed in the frame of the star. The pattern of special directions affects the harmonic content of the X-ray flux time series but not the repetition frequency, which is determined only by the symmetry of the pattern. If the pattern of special directions has n-fold symmetry, the resulting pattern in the X-ray flux repeats with frequency

$$\omega = n\ (\Omega - \Omega_s)\ , \qquad (1)$$

where Ω is the angular velocity of the clump and Ω_s is the angular velocity of the star. We refer to $\Omega - \Omega_s$ as the fundamental beat frequency and $n(\Omega - \Omega_s)$ as the n*th* harmonic of the fundamental. We expect $\Omega \approx \Omega_{K0}$, the Keplerian angular velocity at the radius[7]

$$r_0 = 0.6\ \mu^{4/7}\ (GM)^{-1/7}\ \dot{M}^{-2/7} \qquad (2)$$

of the boundary layer. Here μ and M are the magnetic moment and mass of the star and $\dot{M}$ is the mass flux through the inner edge of the disk, which we assume to be equal to the accretion rate onto the surface of the neutron star.

Consider now a collection of clumps arranged randomly in space but all orbiting within the boundary layer. If the X-ray flux associated with the motion of the i*th* clump within the boundary layer is, for example,

$$F_i(t) = F_{0i} + F_{1i}\ [\ 1 + \cos(\omega_i t + \phi_i)\ ]\ , \qquad (3)$$

where ω_i is the frequency and ϕ_i the phase of the i*th* clump relative to its special direction(s), then the flux contributed by the collection of clumps will vary as

$$F(t) = \sum_{i=1}^{N} F_i\ (t)\ . \qquad (4)$$

The mean flux from the system is then $\langle F(t) \rangle = N\ (F_0 + F_1)$, where $F_0 = \langle F_{0i} \rangle$ and $F_1 = \langle F_{1i} \rangle$.

The power density spectrum $P(f)$ of $F(t)$ has a peak at the mean frequency

$$f_0 = \frac{1}{2\pi} \sum_{i=1}^{N} \omega_i F_{1i}^2 \Big/ \sum_{i=1}^{N} F_{1i}^2 \ , \tag{5}$$

The total power under the peak is $P_{osc} \approx N \langle (F_1/2)^2 \rangle$, corresponding to a variation in the X-ray flux of amplitude $F_{osc} \approx \sqrt{N} \langle F_1 \rangle$. Thus, the fractional modulation produced by the clumps is

$$m = \frac{F_{osc}}{\langle F \rangle} \approx \frac{1}{\sqrt{N}} \frac{F_1}{F_0 + F_1} \ . \tag{6}$$

The production of a peak in the power density spectrum at f_0 is not specific to equation (3), which has been used simply for illustration, but is a very general feature of the model.

Equations (3) and (4) assume that the beat frequency produced by each clump is constant and that the train of pulses lasts indefinitely. In reality, of course, the wave train produced by a given clump lasts only a finite time. Moreover, the frequency of the oscillations produced by a given clump varies in time as the clump drifts inward across the boundary layer, since the angular velocity within the boundary layer varies with radius. These two effects cause the peak in the power spectrum at f_0 to have a finite width. If the lifetime of a clump within the boundary layer is much longer than the beat period, the two effects may be treated separately as follows.

The power density spectrum of the X-ray flux produced by a single clump with a constant angular velocity Ω but a finite lifetime τ is peaked at the beat frequency (1) but has a finite relative width

$$\frac{\Delta f}{f_0} \approx \frac{1}{\omega_0 \tau} \ . \tag{7}$$

The variation of angular velocity with radius within the boundary layer may be taken into account by assuming that at any given time each clump is producing a single, well-defined beat frequency, but that other clumps are producing somewhat different beat frequencies. Then the power spectrum of the X-ray flux produced by all the clumps in the boundary layer is just the incoherent sum of the spectra produced by the individual clumps. Thus, the relative width of the peak at f_0 due to the velocity shear in the boundary layer is

$$\frac{\Delta f}{f_0} \approx \frac{\Delta\Omega}{\omega_0}, \tag{8}$$

where $\Delta\Omega$ is the change in the angular velocity across the boundary layer and ω_0 is the mean beat frequency of clumps in the boundary layer.

This model predicts a systematic variation in the frequency f_0 of the peak in the power density spectrum of the X-ray flux time series as the time-averaged mass flux through the boundary layer varies. For times much shorter than the spin-up time of the neutron star, here ~ 10^5 years, the stellar rotation frequency Ω_S may be treated as essentially constant. Thus, for small changes in the mass flux, the change in f_0 is

$$\Delta f_0 \approx \frac{\partial\omega}{\partial r_0} \frac{\partial r_0}{\partial \dot{M}} \frac{\Delta\dot{M}}{2\pi} \approx \frac{3n}{14\pi} \Omega_{K0} \frac{\Delta\dot{M}}{\dot{M}} . \tag{9}$$

If all X-rays were produced at the surface of the neutron star and if there were no changes in the geometry of the emission region or the X-ray spectrum, then the X-ray flux F observed at earth in a given energy band would simply be proportional to the mass accretion rate $\dot{M}$. However, in GX 5-1 and other bright galactic-bulge sources there are strong indications that both the geometry of the emission region and the spectrum vary with changes in the accretion rate.[8-10] We therefore consider the more general relation $F \propto \dot{M}^{\beta}$.

Using this relation and equations (1) and (8), we can express the logarithmic derivative of the central frequency f_0 with respect to that of the X-ray flux as

$$\alpha = \frac{d\ln\omega_0}{d\ln F} \approx \frac{3}{7\beta}\left(\frac{\Omega_{K0}}{\Omega_{K0} - \Omega_s}\right) . \qquad (10)$$

For small variations in the mass flux, $\alpha \approx$ const.

A direct consequence of our model is that the power density spectrum of the X-ray flux time series contains red noise. The reason is that the X-ray flux contributed by the clumps is positive at all times but is correlated only for time intervals less than the mean time τ_c that a given clump affects the X-ray flux. The total power in the red noise is substantial. For example, in the simple case illustrated by equation (3) and for $\omega_0\tau >> 1$, the total power in the red noise is $\sim N(F_0 + F_1)^2$, while that in the oscillations is $\sim N(F_1/2)^2$. If there are clumps that contribute to the noise but not to the oscillations, the relative power in the red noise will be greater. (The reverse is not possible, since clumps that contribute to the oscillations necessarily contribute to the noise. A related point is that the mean time τ during which a given clump contributes to the oscillations must be less than or comparable to τ_c.) As the power in the oscillations increases (decreases), the power in the red noise must also increase (decrease), although in general the noise power will not be proportional to the power in the oscillations.

Figure 1 shows a sample power density spectrum given by our model, with parameters chosen to give a peak like that reported[1] in GX 5-1. The detailed shape of the spectrum contains a great deal of information about the accretion flow. For example, the observed width of the peak places a lower bound on the mean time τ during which a clump contributes to the oscillating X-ray flux and hence an upper bound on the mean radial velocity v_{r0} in the boundary layer.

Setting $\tau = \delta/v_{r0}$, where δ is the width of the boundary layer, equation (7) interpreted as an inequality gives

$$\frac{v_{r0}}{v_{K0}} \lesssim \frac{\delta}{r_0} \cdot \frac{\omega_0}{\Omega_{K0}} \cdot \frac{\Delta f}{f_0} , \tag{11}$$

where v_{K0} is the Keplerian velocity at r_0. The shape of the red noise component in the spectrum reveals the time history of the X-ray flux contributed by an individual clump. Moreover, the noise spectrum flattens below the critical frequency $f_c \approx 1/2\pi\tau_c$, which yields an estimate of τ_c. Furthermore, the Fourier amplitudes of the oscillating and mean fluxes contributed by each clump interfere. This interference affects the shape of the peak in the power spectrum as well as the nearby continuum, providing further information about how a clump affects the X-ray flux emitted by the system.

Physical Mechanisms

Having described the general features of our model, we now discuss briefly specific physical mechanisms that could cause clumping in the inner disk and modulation of the X-ray flux at the frequency (1). A more complete discussion will be given elsewhere.[11]

There are several factors that make clumping of the flow in the inner disk highly likely in systems like GX 5-1. First, in such weakly magnetic neutron stars the disk-magnetosphere boundary lies only a few stellar radii from the surface of the star. At such small radii the disk is thermally unstable to separation into denser, colder clumps and a hotter inter-clump medium.[12-15] Second, the relative motion of the magnetospheric and disk plasma drives the Kelvin-Helmholtz instability to large amplitudes near the inner edge of the disk, where the disk plasma is partially confined by the pressure of the magnetospheric field.[6,16,17] Because the radiation flux from the surfaces of the inner disk is comparable to the critical flux there, the geometrical thickness of the inner disk is significantly increased by radiation pressure. More detailed considerations suggest a picture in which the hotter plasma in the disk extends to a height $h \gtrsim 1.5(R/r_0)r_0 \sim 0.3r_0$, whereas the clumps have a vertical scale height $h_c << h$. Here R is the radius of the neutron star.

How do the clumps in the inner disk modulate the X-ray flux from the system? There are a variety of possible mechanisms. Here we mention two. First, if the magnetosphere is not axisymmetric in the disk plane (as will be the case if the magnetic axis of the star is not aligned with its rotation axis, or if the magnetic axis is aligned but offset with respect to the center of the star), the interaction of a given clump with the stellar magnetic field will be greater at some azimuthal positions within the boundary layer than at others. As a result, the plasma entry rate, and hence the radiation from the surface of the neutron star, will be greater when the clump is at some stellar azimuths than at others. The entry rate may be greater because of enhanced reconnection of magnetic flux in the clump with the stellar flux or simply because the magnetospheric field configuration is more favorable for plasma entry. Second, if the pattern of radiation from the surface of the neutron star is not axisymmetric in the disk plane, a given clump will scatter more of this radiation into the line of sight at some stellar azimuths than at others. For a variety of reasons, we expect this effect to be less important than modulation of the mass flux to the stellar surface.

We refer to the stellar azimuth or azimuths at which the X-ray flux from the system is enhanced for a given clump as the special direction(s) for that clump (see Fig. 2). Note that equations (3) and (4) hold even if the special directions for each clump are different, so long as they remain fixed in the frame of the star. The symmetry of the pattern of special directions depends on the stellar field pattern and the nature of the modulation mechanism. If the star's magnetic axis is tilted with respect to its rotation axis and plasma attaches to the stellar field via reconnection, for example, $\underline{n} = 1$ if the orientation of the magnetic field in the clumps remains fixed in the frame of the star, whereas $\underline{n} \approx 2$ if the orientation remains fixed in the inertial frame. If instead the magnetic axis is aligned but offset, $\underline{n} = 1$. For scattering of radiation from the stellar surface by the clumps, one expects $\underline{n} = 2$.

The harmonic content of the oscillations also provides important information about the physics of the boundary layer. We have calculated the expected harmonic content for several models of the interaction of clumps with the magnetosphere. These calculations indicate that if r_0 is large enough compared to the stellar radius that the magnetic field at the boundary layer is nearly dipolar, the power in one of the harmonics of the basic beat frequency $\Omega - \Omega_S$ dominates that in all the others by a factor ~ 20–100, owing to the smoothness of the dipole field pattern. Accurate measurement of the harmonic content of the oscillations can help to determine the nature of the disk-magnetosphere interaction.

If the quasiperiodic oscillations are due to modulation of the mass flux to the stellar surface, as we have suggested, both the oscillating and the red noise in the X-ray flux should be most prominent in the so-called 'hard' component of the X-ray spectrum.

Absence of Periodic Pulsations

Why has it been so difficult to detect periodic X-ray pulsations at the rotation frequencies of the neutron stars in the bright galactic-bulge X-ray sources? The canonical view is that the neutron stars in these sources have no appreciable magnetic fields (see, e.g., ref. 18). Obviously, the present interpretation of the quasiperiodic oscillations in GX 5-1 is inconsistent with this view. We suggest instead that the magnetic fields of neutron stars in the bright bulge sources span a wide range, from $< 10^8$ G to $\sim 10^{11}$ G. The magnetospheres of stars with the weakest fields do not channel the accreting plasma appreciably and hence do not produce collimated X-rays. However, the magnetospheres of stars with fields near the upper end of this range surely do channel the accretion flow[19] and therefore could produce an X-ray beam. Nevertheless, there are a variety of factors present that by themselves or by interaction with one another are likely to make X-ray pulsations at the rotation frequencies of these stars difficult to detect.

First, for the smaller magnetospheric radii characteristic of fields $\ll 10^{12}$ G the inner disk is unstable and may have a relative vertical thickness h/r_0 substantially greater than the inner disks around stars with stronger fields. Second, there is compelling evidence that optically thick accretion disk coronae are formed at the high luminosities of the brightest bulge sources.[20,21] Both factors make observation of X-rays coming directly from the neutron star difficult except for systems with relatively small inclinations.[18] Assuming that the rotation axis of the neutron star is parallel to that of the disk, this means that radiation coming directly from the neutron star will be observed only for viewing angles close to the star's rotation axis, where modulation due to beaming is suppressed.

Third, the degree of X-ray beaming in the bright galactic-bulge sources may be much less than in canonical accretion-powered pulsars, owing to the high luminosities and relatively weak magnetic fields of the former. As a result of their high luminosities, the radiation pressure within the magnetospheres of the bright bulge sources may be sufficient to cause accreting plasma to settle over a large fraction of the neutron star's surface.[10] The relatively weak magnetic fields of these neutron stars act in the same direction, since weak fields are generally less effective in channeling the accreting plasma.[19] Observational evidence that accreting plasma falls over a substantial fraction of the stellar surface comes from spectral fits, which yield emitting areas comparable to the surface area of a neutron star.[10,22] Such a broad distribution of accreting plasma may be expected to produce a relatively broad beam with reduced modulation.

Fourth, to the extent that the X-rays emitted from the neutron stars in the bright bulge sources are beamed, they may be beamed into the plane of the disk. Observational support for such beaming comes from the modest luminosities of the so-called 'hard' components in the X-ray spectra of these sources when compared to the luminosities of the so-called 'soft' components[8-10] and from the evidence for accretion disk coronae produced by evaporation of plasma from the disk by X-rays from the neutron star.[20] If the magnetic axis is perpendicular to the rotation axis, this would imply pencil beaming. If instead the magnetic axis is aligned but offset from the rotation axis, this would imply a fan beam. In this latter case the alignment itself would suppress modulation of the X-ray flux at the stellar rotation frequency.

Fifth, radiation pressure and evaporative heating associated with the very high luminosities of some of the bulge sources may create a relatively dense central corona around the neutron star,[21] causing X-ray pulsations at the stellar rotation frequency to be washed out.[23]

Quasiperiodic X-ray oscillations produced according to our model are likely to survive better in the presence of substantial circumstellar plasma than pulsations at the stellar rotation frequency for two reasons. First, the oscillation frequency is typically much smaller than the stellar rotation frequency and so is much less affected by time-of-flight smearing. Second, the X-ray oscillations arise from variations in the mass flux onto the neutron star. Thus, even if the emission from the neutron star were perfectly isotropic, the oscillations would still be observable. As discussed above, the emission from the neutron star may in fact be rendered isotropic by scattering in an optically thick circumstellar corona. In this case the oscillations will be observable if the mean time for photons to propagate through the corona is less than the oscillation period. In contrast, periodic pulsations at the stellar rotation frequency arise not from variations in the mass flux, but from angular anisotropy in the pattern of emission from the stellar surface. Scattering that isotropizes the radiation from the neutron star will destroy such pulsations completely.

If our analysis of the reasons for the absence of prominent X-ray pulsations at the rotation frequency of the neutron star is substantially correct, the probability of observing the underlying rotation period should be highest in relatively low-luminosity sources.

Application to GX 5-1 and Other Sources

Having described our model in some detail, let us now apply it specifically to GX 5-1. Quasiperiodic oscillations in the 1-10 keV X-ray flux have been observed using EXOSAT.[1] The frequency of the oscillations was 20 Hz when the count rate was 2400 counts s^{-1} and 40 Hz when the count rate was 3100 counts s^{-1}. The relative width of the peak in the power density spectrum at the frequency of the oscillations was $\Delta f/f_0 \sim 0.25$. The reported value of α (cf. eq. [10]) is ~ 2.5. In addition, red noise is present in the X-ray flux time series with a strength that increases with the strength of the oscillations.[4]

Equation (1) gives a variation in the frequency of the quasiperiodic oscillations with X-ray flux like that reported, for a substantial range of neutron star magnetic fields and stellar spin rates. For small variations in the mass flux, comparable to the variations in the X-ray flux observed while oscillations are present, the value of α given by equation (10) is $\approx$ const. Of course, for large variations in the mass flux, α varies. Thus, a key question for the model is the extent to which the observed variation in the time-averaged X-ray flux is due to variation in the average mass accretion rate as opposed to variation in the geometry of the emitting region or regions at the surface of the neutron star. A second key question is which harmonic of the fundamental beat frequency is being observed.

Six possibilities are illustrated in Table 1. Models 1-3 assume that the fundamental beat frequency (n=1) is being observed, whereas Models 4-6 assume that the observed frequency is the first overtone (n=2). All models assume that the so-called 'soft' component of the spectrum is produced by the accretion disk whereas the 'hard' component is produced near the surface of the neutron star.[8-10] In Models 1 and 4 the mass flux onto the neutron star changes by only a small amount. Thus, the observed changes in intensity are due mostly to changes in the projected area of the emitting region(s). In Models 2 and 5 the mass flux and the projected area both change by a moderate amount, whereas in Models 3 and 6 the change in the X-ray flux is due entirely to change in the mass flux. The neutron star surface magnetic field B_s and rotation frequency f_s were adjusted to give the observed values of f_0 and Δf_0. The neutron star rotation rates and magnetic field strengths inferred from our model for GX 5-1 lend support to the idea that systems like it are progenitors of the millisecond rotation-powered pulsars.[3,24-26]

Table 1 shows that the harmonic number $\underline{n}$ and the change in the mass flux can be determined immediately if the underlying rotation frequency of the neutron star is detected. If it is not, one can still determine the harmonic number and estimate the change in the mass flux by studying the shape of the oscillation frequency vs. X-ray flux curve, which is slightly different in each of the six cases.

The reported[1] width $\Delta f \sim 0.25 f_0$ of the peak in the power spectrum implies (cf. eq. [7]) that a typical clump contributes to the oscillations for at least 4-5 oscillation periods. This in turn implies that the radial drift velocity in the boundary layer is much less than free-fall (cf. eq. [11]) and hence that the angular velocity there is close to Keplerian. It is interesting to note that if the width δ of the boundary layer is comparable to the vertical scale height h_c of the clump distribution in the boundary layer, then the lower bound on the time spent by a clump in the boundary layer given by Δf implies $\delta \gtrsim 0.02 r_0$. The shear in the Keplerian angular velocity across $0.02 r_0$ then gives $\Delta\omega \sim 0.25\omega_0$. Thus, the width of the peak may be due primarily to the shear in the boundary layer. If this is the case, the shoulder in the red noise component of the power spectrum lies well below f_0. A bound on the mean lifetime of a clump derived from the shape of the red noise component would place powerful constraints on the physics of the boundary layer.

As well as interpreting the quasi-periodic oscillations, the present model also suggests a natural explanation for the 'dips' occasionally seen[10] in GX 5-1 when the time-averaged X-ray intensity is near its minimum value. In our model the radius r_0 of the inner edge of the disk is quite close to the corotation radius r_c. Thus, when the mass accretion rate is near its minimum value, r_0 may briefly equal or exceed r_c. When this happens, the mass flux from the inner edge of the disk to the neutron star temporarily ceases[19,6] and one observes only the 'soft' component from the disk.[10]

The disappearance of X-ray oscillations with increasing X-ray flux reported[1] in GX 5-1 may be due in part to the fact that conditions can be more favorable for generating large clumps via the Kelvin-Helmholtz instability when the inner radius r_0 of the disk is not too far inside the corotation radius r_c.[15] If these conditions are met and if clumping is triggered by the Kelvin-Helmholtz instability, the degree of clumping in the boundary layer will decrease as the mass flux through the boundary layer increases, since this causes r_0 to move inward from r_c. In addition, as the mass flux increases, causing the flux of radiation from the neutron star and from the surfaces of the disk to increase, the height and density of the accretion disk corona is likely to grow. If so, the oscillations could disappear as a result of scattering and absorption of the radiation from the neutron star by the disk corona. In this case, changes in the X-ray spectrum characteristic of the development of a denser and more extensive corona[23] should be observable. Accurate measurements of the X-ray spectrum and of the total luminosity of the central source can help to determine the relative importance of X-ray heating and radiation pressure.

We have already suggested why detection of X-ray pulsations at the rotation frequency of the neutron star is likely to be difficult. If our analysis is correct, detection of pulsations is likely to be especially difficult in GX 5-1 for two reasons. First, there is strong evidence that the inclination angle of GX 5-1 is quite small[10] and hence that our viewing angle is very near the rotation axis. Second, GX 5-1 is a very high-luminosity bulge source which is therefore likely to have a particularly dense and extensive accretion disk corona.

In closing, we emphasize that the phenomena described here may be observable in other galactic bulge X-ray sources. We note that quasiperiodic oscillations similar in character to those observed in GX 5-1 have recently been reported in Sco X-1.[27] The 2 Hz oscillations seen[28] in long flat-top bursts from the Rapid Burster may also be a similar phenomenon. If so, our model implies that the Rapid Burster has a surface magnetic field ~ 6×10^{10} gauss. If the 10^{-3} Hz oscillations reported[29] in the 8 s accretion-powered pulsar 4U1626-67 also prove to be an example of this phenomenon,[3] it would demonstrate that the thermally unstable region of the inner disk is not essential to the development of clumping, since such a region is absent in this source. Our model then gives an estimate of the surface magnetic field of ~ 4×10^{12} gauss. This estimate is independent of the similar estimate derived from the observed spin-up rate,[30] which is consistent with it. Finally, we note that the present model, when scaled appropriately, may explain some of the higher-frequency quasiperiodic oscillations observed in some cataclysmic variables, such as AE Aqr.[31,32]

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TABLE 1

REPRESENTATIVE MODELS OF GX 5-1*

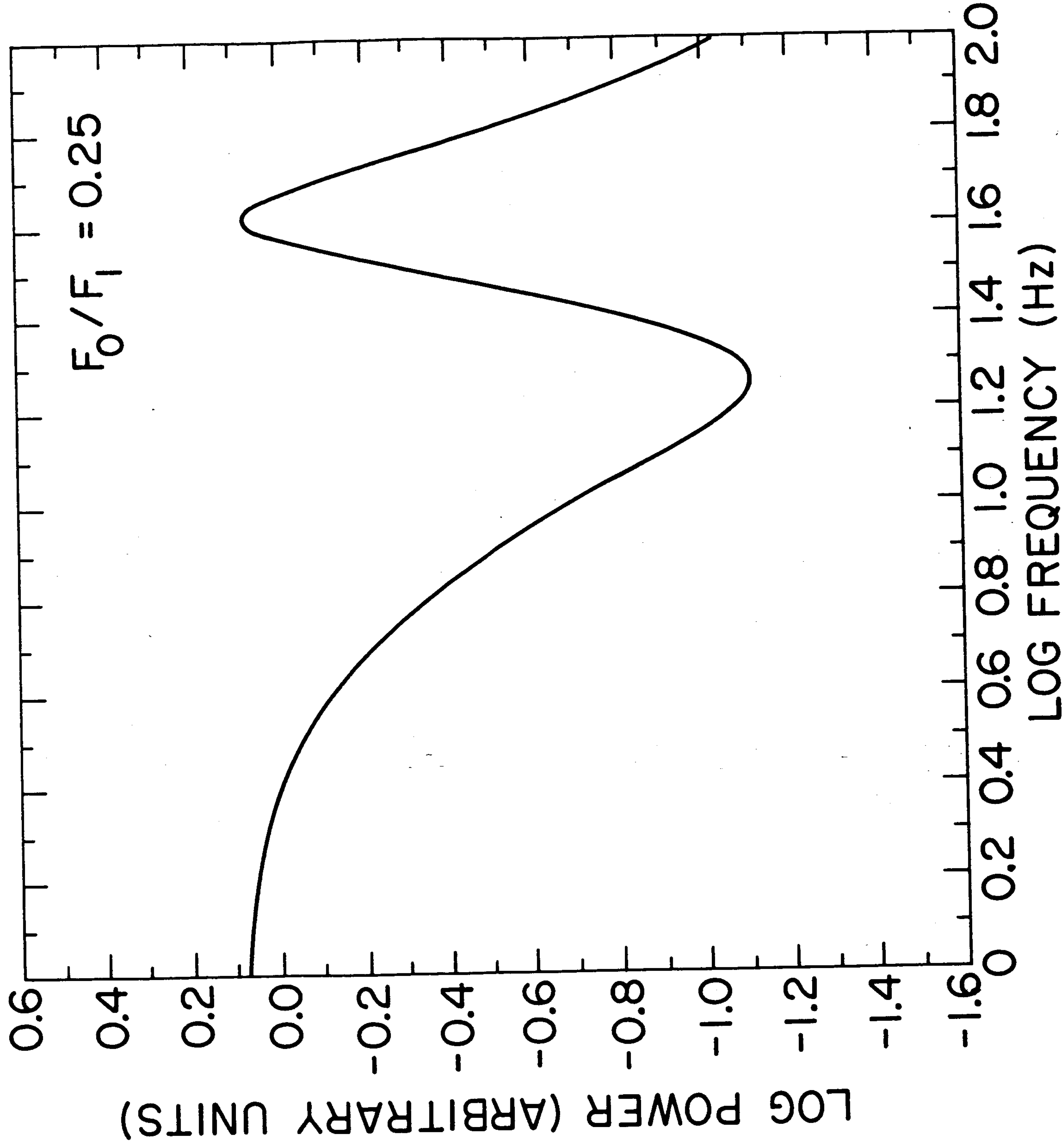
MODEL	n	B_s	f_s	P_s	$\dot{M}_{min}$	$\dot{M}_{max}$	r_0
1	1	3.4	458	2.2	0.50	0.55	2.7
2	1	11.7	148	6.8	0.50	0.65	5.1
3	1	19.4	73	13.6	0.35	0.55	7.2
4	2	7.7	229	4.4	0.50	0.55	4.2
5	2	26.3	74	13.6	0.50	0.65	8.1
6	2	43.5	37	27.2	0.35	0.65	11.3

*All models assume a 1.4 $M_\odot$ neutron star of radius 10^6 cm. The symbols and their units are: harmonic number n, stellar magnetic field B_s (10^9 G), rotation frequency f_s (Hz), rotation period P_s (ms), minimum mass flux $\dot{M}_{min}$ ($\dot{M}_E$), maximum mass flux $\dot{M}_{max}$ ($\dot{M}_E$), and boundary layer radius r_0 (10^6 cm) at the maximum mass flux. Here $\dot{M}_E$ is the mass flux corresponding to an accretion luminosity of 1.76×10^{38} ergs s^{-1}.

FIGURE CAPTIONS

Fig. 1 Sample power density spectrum of the X-ray flux time series given by the present model, showing the red noise and quasiperiodic oscillations. Parameters have been chosen to give a peak at 40 Hz with a relative width $\Delta f/f_0 \sim 0.25$, as observed in GX 5-1 at 2400 counts s^{-1}. In this example half the width of the peak is due to lifetime (homogeneous) broadening and half to velocity shear (inhomogeneous) broadening.

Fig. 2 Schematic drawing of the narrow boundary layer at the inner edge of the disk, from which plasma accretes along field lines onto the surface of the neutron star. The radius and width of the boundary layer are r_0 and δ respectively. The spiral depicts the trajectory of a given clump, as seen in a frame corotating with the star. In this frame, the clump moves with mean azimuthal velocity $r\omega(r)$ and mean radial velocity v_{r0}. The clump loses matter to the magnetosphere continuously, but the rate of this transfer depends on the azimuthal position of the clump. The sections of the trajectory where the transfer rate is high for this particular clump (the 'special directions' referred to in the text) are depicted with heavy lines whereas those where it is low are depicted with light lines. In this example, the frequency of the observed oscillations would be equal to the fundamental beat frequency $\Omega - \Omega_s$. If instead the pattern of special directions has two-fold symmetry, the transfer rate is also high along those sections of the trajectory indicated with dashed heavy lines, and the frequency of the observed oscillations is the first overtone of the fundamental beat frequency.



$F_0/F_1 = 0.25$
LOG FREQUENCY (Hz)
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
LOG POWER (ARBITRARY UNITS)
0.6 0.4 0.2 0.0 -0.2 -0.4 -0.6 -0.8 -1.0 -1.2 -1.4 -1.6

Figure 1

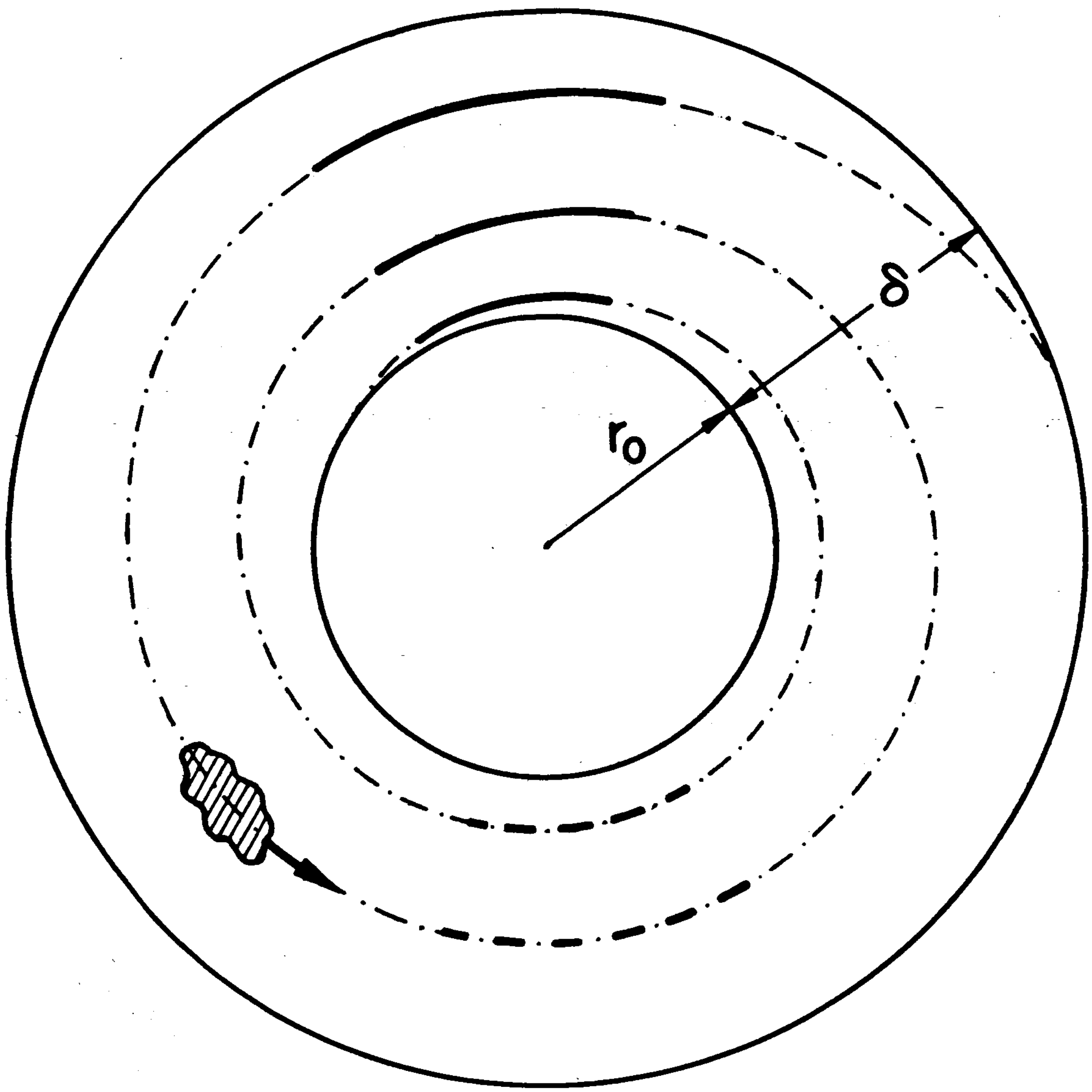


Figure 2